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Interval Goal Programming Approach to Multiobjective Fuzzy Goal Programming Problem with Interval Weights

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Abstract

This article presents interval goal programming (IGP) approach for solving linear multiobjective fuzzy goal programming problem with interval weights. In the proposed approach, interval weights for achievement of fuzzy goals to their aspired levels on the basis of their relative importance are considered in an uncertain environment.

In the model formulation of the problem, the membership functions for each of the fuzzy goals are defined first. Then, the membership functions are transformed into membership goals by assigning the highest membership value (unity) and introducing under-and over-deviational variables to each of them.

In the solution process, the interval weights (derived from pairwise interval judgment matrix) associated with the unwanted deviational variables is introduced in the goal achievement function for minimizing them to reach the aspired goal levels of the problem.

To illustrate the proposed approach, a numerical example is solved.

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Keywords: Interval arithmetic, interval programming, pairwise comparison matrix, fuzzy programming, fuzzy goal programming, goal programming.

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1. Introduction

Most of the real-world problems are multiobjective in nature and the objectives conflict to each other. To resolve the conflict, goal programming (GP) approach has been introduced by Charnes and Cooper [1] in 1961. But, in most of the cases, goal values for the different objectives cannot be defined precisely. To tackle such an imprecision, fuzzy programming (FP) approach in the area of multiobjective linear optimization based on fuzzy set theory [2], has been introduced by Zimmermann [3] in 1978. In FP, a membership function is defined on the basis of aspiration levels and the tolerance limits. Then, maxmin approach is used to reach the desired solution. In some of the cases, tolerance limits cannot be defined in highly sensitive decision situation. To resolve the problem, GP approach in fuzzy environment has been first introduced by Narasimhan [4] in 1980. Thereafter, FGP has been extensively studied by Hannan [5], Tiwari et al. [6], Pal et al. [7], and applied in different real -life problems in [8, 9].

In multiobjective optimization, relative importance of one objective over another is defined as weight of the first objective. The weights play an important role for achieving the solution of the multiobjective programming problem according to the needs and desires of the decision makers (DMs). Again, in GP, weights associated with the unwanted deviational variables measure the relative importance of the respective objective .The different methodologies to derive the weights or priorities have been studied by Pekelman and Sen [10], Srinivasan [11], Chen and Tsai [12] in the past. The fuzzy weights have been employed for solving multiobjective fuzzy fractional programming problem by Pal et al. [7] in 2003. All the methodologies studied previously in the area of GP or FGP, weights of relative importance are defined as crisp values. But it is more realistic to consider the weights as in interval form.

The interval programming approach is a prominent tool for solving the multiobjective programming problem involved with interval uncertainty. The interval programming, based on interval arithmetic was introduced by Moore [13] in 1962. The interval programming approach in the area GP has been introduced by Inuguichi and Kume [14] in 1991. The different methodologies studied in the past have been surveyed by Oliveira and Antunes [15].

The idea of uncertainty in the weights structure has been introduced by Saaty and Vargas [16] in 1987. The priorities determined from pairwise interval comparison matrix have been suggested by Sugihara et al. [17]. Determination of interval weights from interval comparison matrix has been proposed by Wang and Elhag [18] in 2007. But, the interval weights associated with unwanted deviational variables in the area of GP or FGP, yet to be circulated in the literature. To tackle such uncertain weight structure, in the proposed approach, weights associated with unwanted deviational variable in goal achievement function have been taken as in interval form. Then, IGP methodology is the appropriate technique to solve such type of problem.

In this paper, IGP approach has been presented to solve the fuzzy multiobjective programming problem with interval weights associated to the goal achievement function. According to the defined aspiration levels and lower tolerance range, membership functions of the defined fuzzy goals are constructed. The attainable highest membership value of the membership function is unity. Considering the target value as unity for each of the membership function and introducing under-and over-deviational variables, membership goals are constructed.

The goal achievement function is addressed as the weighted sum of unwanted deviational variables. The weights (i.e. interval form) are determined by using GP methodology [15] with the help of pairwise interval judgment matrix. Then, the problem is the form of an interval programming problem. Using the IGP approach [17], interval goals are transformed to standard goals. To construct the regret function of the final executable model, the sum of unwanted deviations associated with the respective goals is taken into consideration to achieve the goal values in the specified range. Then the problem is solved by using standard GP methodology.

To illustrate the proposed approach a numerical example is solved.

2. Model Formulation

In the most of the practical situation, targets are imprecisely defined. Then the goals are fuzzily defined.

The generic form of the fuzzy goal programming problem can be presented as:

$$Z_k(X) \gtrsim b_k, \quad k = 1,2,\ldots,K_1.$$

$$Z_k(X) \lesssim b_k, \quad k = (K_1+1),(K_1+1),\ldots,K$$

subject to $X \in S = \{X \in R^n \mid AX \begin{pmatrix} \geq \\ \leq \end{pmatrix} C,\ X \geq 0, C \in R^m\}$ (1)

Now, description of fuzzy goals is defined as follows:

1.1 Construction of Membership Goals

Let b_k be the imprecise aspiration level of the k-th objective $Z_k(X)$ (k=1, 2,….,K). Then the fuzzy goals take either of the form $Z_k(X) \gtrsim b_k$ or $Z_k(X) \lesssim b_k$ depending on the maximizing or minimizing the objectives, where X is the vector of decision variables, $\gtrsim$ and represent the fuzziness of $\geq$ and $\leq$ restrictions respectively, in the sense of Zimmermann [3].

In a decision situation, fuzzy goals are characterized by their respective membership functions.

The membership function for $Z_k(X) \gtrsim b_k$ appear as:

$$\mu_k(X) = \begin{cases} 1 & \text{if } Z_k(X) \geq b_k, \\ \dfrac{Z_k(X) - l_k}{b_k - l_k} & \text{if } l_k \leq Z_k(X) < b_k, \\ 0 & \text{if } Z_k(X) < l_k, \end{cases} \qquad k = 1, 2, ..., K_1 \tag{2}$$

Again, for type of restriction, $Z_k(X) \lesssim b_k$, the membership function takes the form

$$\mu_k(X) = \begin{cases} 1 & \text{if } Z_k(X) \leq b_k, \\ \dfrac{u_k - Z_k(X)}{u_k - b_k} & \text{if } b_k < Z_k(X) \leq u_k, \\ 0 & \text{if } Z_k(X) > u_k, \end{cases} \qquad k = (K_1 + 1), (K_1 + 2), ..., K. \tag{3}$$

Then the membership goals of the defined membership functions with highest membership value (unity) appear as:

$$\frac{Z_k(X) - l_k}{b_k - l_k} + \eta_k^- - \eta_k^+ = 1, \qquad k = 1, 2, ..., K_1 \tag{4}$$

and $$\frac{u_k - Z_k(X)}{u_k - b_k} + \eta_k^- - \eta_k^+ = 1, \qquad k = (K_1 + 1), (K_1 + 1), ..., K \tag{5}$$

where $\eta_k^+, \eta_k^- \geq 0$ represent the under- and over-deviational variables concerned with achievement of the aspired level of the k-th membership goal.

3. Preliminaries of Interval Arithmetic

Let a closed interval A (called an interval number) is defined by
$A = [a^L, a^U] = \{a: a^L \leq a \leq a^U, a \in \Re\}$, where a^L, a^U are left and right limits, respectively, of the interval A on the real line $\Re$.

For a particular case, A = [a, a] represents only the real number a.

Now, the different interval arithmetic operations are defined as follows:

The binary operation addition '+' between two interval numbers $A_1 = [a_1^L, a_1^U]$ and $A_2 = [a_2^L, a_2^U]$ is defined as:

$$A_1 + A_2 = [a_1^L + a_2^L, a_1^U + a_2^U]$$

The multiplication of two interval number A_1 and A_2 is defined as:

$$A_1 * A_2 = [\min\ (a_1^L a_2^L, a_1^L a_2^U, a_1^U a_2^L, a_1^U a_2^U),\ \max\ (a_1^L a_2^L, a_1^L a_2^U, a_1^U a_2^L, a_1^U a_2^U)]$$

Division of two interval numbers is defined as:

$$A_1 / A_2 = [\min\ (\frac{a_1^L}{a_2^L}, \frac{a_1^L}{a_2^U}, \frac{a_1^U}{a_2^L}, \frac{a_1^U}{a_2^U}), \max\ (\frac{a_1^L}{a_2^L}, \frac{a_1^L}{a_2^U}, \frac{a_1^U}{a_2^L}, \frac{a_1^U}{a_2^U})]$$

provided $a_2^L, a_2^U \neq 0$.

For particular case, when $(a_1^L, a_1^U, a_2^L, a_2^U) > 0$

then, $A_1 / A_2 = [\frac{a_1^L}{a_2^U}, \frac{a_1^U}{a_2^L}]$.

4. Determination of Interval Weights

Weights of importance of unwanted deviational variable are used to represent the relative importance of the respective criteria. It is more realistic to measure the relative importance in interval form rather than the deterministic values.

If $[w_i^L, w_i^U]$ (where $w_i^L, w_i^U > 0$) be interval weight of importance of the objective Z_i, and also the pairwise judgments are precise, then interval comparison matrix A can be presented as:

$$A = \begin{pmatrix} 1 & \frac{[w_1^L, w_1^U]}{[w_2^L, w_2^U]} & \cdots & \frac{[w_1^L, w_1^U]}{[w_n^L, w_n^U]} \\ \frac{[w_2^L, w_2^U]}{[w_1^L, w_1^U]} & 1 & \cdots & \frac{[w_2^L, w_2^U]}{[w_n^L, w_n^U]} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{[w_n^L, w_n^U]}{[w_1^L, w_1^U]} & \frac{[w_n^L, w_n^U]}{[w_2^L, w_2^U]} & \cdots & 1 \end{pmatrix}$$

where $\frac{[w_i^L, w_i^U]}{[w_j^L, w_j^U]}$ represents the relative importance of Z_i over Z_j.

Using interval arithmetic defined in the Section 3, the interval comparison matrix can be expressed as:

$$A = \begin{pmatrix} 1 & [\frac{w_1^L}{w_2^U}, \frac{w_1^U}{w_2^L}] & \cdots & [\frac{w_1^L}{w_n^U}, \frac{w_1^U}{w_n^L}] \\ [\frac{w_2^L}{w_1^U}, \frac{w_2^U}{w_1^L}] & 1 & \cdots & [\frac{w_2^L}{w_n^U}, \frac{w_2^U}{w_n^L}] \\ \cdots & \cdots & \cdots & \cdots \\ [\frac{w_n^L}{w_1^U}, \frac{w_n^U}{w_1^L}] & [\frac{w_n^L}{w_2^U}, \frac{w_n^U}{w_2^L}] & \cdots & 1 \end{pmatrix} \tag{6}$$

If (i, j) th element of the matrix defined in (6), is designated by $[l_{ij}, u_{ij}]$ then $l_{ij} = \frac{w_i^L}{w_j^U}$, $u_{ij} = \frac{w_i^U}{w_j^L}$ and obviously,

$$l_{ij} \times u_{ji} = 1 \qquad \text{for } i, j = 1, 2, \ldots, n. \tag{7}$$

Again, the two relations

$$A_L W_U = W_U + (n-1) W_L \tag{8}$$

and

$$A_U W_L = W_L + (n-1) W_U \tag{9}$$

are satisfied where

$$A_L = \begin{pmatrix} 1 & \frac{w_1^L}{w_2^U} & \cdots & \frac{w_1^L}{w_n^U} \\ \frac{w_2^L}{w_1^U} & 1 & \cdots & \frac{w_2^L}{w_n^U} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{w_n^L}{w_1^U} & \frac{w_n^L}{w_2^U} & \cdots & 1 \end{pmatrix} \quad \text{and} \quad A_U = \begin{pmatrix} 1 & \frac{w_1^U}{w_2^L} & \cdots & \frac{w_1^U}{w_n^L} \\ \frac{w_2^U}{w_1^L} & 1 & \cdots & \frac{w_2^U}{w_n^L} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{w_n^U}{w_1^L} & \frac{w_n^U}{w_2^L} & \cdots & 1 \end{pmatrix}$$

and W_L and W_U represent the lower and upper weight vector defined as $W_L = [w_1^L, w_2^L, ..., w_n^L]^T$ and $W_U = [w_1^U, w_2^U, ..., w_n^U]^T$.

But in practical situation, pairwise comparison judgment is not cent percent correct and obviously the relation (7) is not satisfied. Consequently, the relations (8) and (9) are also not satisfied. There are some errors occurred.

If E_1, E_2 be error occurred in satisfying the relations (8) and (9) then the error can be expressed as

$$E_1 = (A_L - I)W_U - (n-1)w_L$$
$$E_2 = (A_U - I)W_L - (n-1)w_U$$

Our goal is to achieve the weights W_L and W_U in such a way that error is to be zero.

Then considering the target values as zero, the goal expression can be written as:

$$(A_L - I)W_U - (n-1)W_L + d_1^- - d_1^+ = 0 \tag{10}$$
$$(A_U - I)W_L - (n-1)W_U + d_2^- - d_2^+ = 0 \tag{11}$$

where $d_i^-, d_i^+ \ (i = 1,2)$ represent the vector of deviational variables of the dimension same as W_L and W_U .

Since we have the target is to achieve the exact value zero, sum of the both under and over deviations associated with the respective goals have to be minimized.

The executable GP model can be expressed as [18]:

$$\text{Minimize} \quad Z = \sum_{i=1}^{2}\sum_{j=1}^{n}(d_{ij}^- + d_{ij}^+)$$

so as to satisfy the goal equations in (10) and (11) and satisfy

$$W_i^L + \sum_{\substack{j=1 \\ j \neq i}}^{n} W_j^U \geq 1 \ , \quad W_i^U + \sum_{\substack{j=1 \\ j \neq i}}^{n} W_j^L \leq 1 \qquad (i = 1,2,...,n)$$
$$W_U - W_L \geq 0,$$
$$W_U, W_L \geq 0. \tag{12}$$

5. FGP Model with Interval Weights

Using interval weights determined from the relations in (12), the goal achievement function associated with the fuzzy goals defined in (4) and (5) can be presented as

$$\text{Minimize } Z = \sum_{k=1}^{K}[w_k^L, w_k^U]\eta_k^- \tag{13}$$

Using interval arithmetic the expression in (12) expressed as:

$$\sum_{k=1}^{K}[w_k^L, w_k^U]\eta_k^- = [\sum_{k=1}^{K} w_k^L \eta_k^-, \sum_{k=1}^{K} w_k^U \eta_k^-]$$

$$= [T_{1L}(\eta^-), T_{1U}(\eta^-)] \text{ (say)} \tag{14}$$

To determine the target interval, individual least solution of $T_{1U}(\eta^-)$ is to be determined first. If $T_{1U}^{\bullet}$ be the minimum value of the function $T_{1U}(\eta^-)$, then $T_{1U}^{\bullet} = \min_{x \in S_1} T_{1U}(\eta^-)$, where S_1 is the feasible region with satisfaction of the goal constraints in (4), (5) and system constraints in (1).

Now achieving the least value of target, the feasible interval $[t_1^L, t_1^U]$ can be taken as:

$$0 \le t_1^L \le t_1^U \le T_{1U}^{\bullet}$$

Incorporating the target interval, the interval objective in (13) can be represented as

$$[T_{1L}(\eta^-), T_{1U}(\eta^-)] = [t_1^L, t_1^U] \tag{15}$$

In GP framework, objectives are transformed into goals by incorporating certain aspiration levels. To achieve the objective values in the target interval $[t_1^L, t_1^U]$, the goals can be expressed as $T_{1L}(\eta^-) \ge t_1^L$ and $T_{1U}(\eta^-) \le t_1^U$. Using under- and over-deviational variables, the goal expressions can be expressed as:

$$T_{1L}(\eta^-) + \rho_{1L}^- - \rho_{1L}^+ = t_1^L,$$

$$T_{1U}(\eta^-) + \rho_{1U}^- - \rho_{1U}^+ = t_1^U \tag{16}$$

where $(\rho_{1L}^-, \rho_{1L}^+) \ge 0$ and $(\rho_{1U}^-, \rho_{1U}^+) \ge 0$.

6. Interval GP Formulation

To achieve the interval goal in the specified interval $[t_1^L, t_1^U]$, sum of under deviation (associated with first goal in (16)) and over deviation (associated with second goal) is to be minimized.

Then, the goal achievement function termed as regret function can be written as Z=$(\rho_{1L}^- + \rho_{1U}^+)$.

The problem can be formulated as

Minimize Z=$(\rho_{1L}^- + \rho_{1U}^+)$

so as to satisfy

$$T_{1L}(\eta^-) + \rho_{1L}^- - \rho_{1L}^+ = t_1^L,$$

$$T_{1U}(\eta^-) + \rho_{1U}^- - \rho_{1U}^+ = t_1^U,$$

$$\frac{Z_k(X) - l_k}{b_k - l_k} + \eta_k^- - \eta_k^+ = 1, \qquad k = 1,2,\ldots,K_1$$

$$\frac{u_k - Z_k(X)}{u_k - b_k} + \eta_k^- - \eta_k^+ = 1, \qquad k = (K_1+1),(K_1+2),\ldots,K \tag{17}$$

subject to the set of system constraints in (1).

7. Numerical Example

To illustrate the proposed approach the following example is considered.

Maximize $Z_1 = 70x_1 - 30x_2$

Maximize $Z_2 = 3x_1 + 8x_2$

Maximize $Z_3 = -4x_1 + x_2$

subject to $2x_1 + x_2 \ge 8,$

$$x_1 + x_2 \geq 5,$$
$$x_1 - 2x_2 \geq -6,$$
$$5x_1 - 2x_2 \leq 18,$$
$$x_1, x_2 \geq 0.$$

The aspiration levels of the three objective goals are taken as 250, 66, - 4 and lower tolerance limits are considered as 20, 20, - 18 respectively.

Now, Membership goals associated with the objectives can be expressed as:

$$(1/230)(70x_1 - 30x_2 - 20) + \eta_1^- - \eta_1^+ = 1$$
$$(1/46)(3x_1 + 8x_2 - 20) + \eta_2^- - \eta_2^+ = 1$$
$$(1/14)(-4x_1 + x_2 + 18) + \eta_3^- - \eta_3^+ = 1 \qquad (18)$$

To formulate the executable model the weights have to be determined.

The set of alternatives between which the pairwise comparison are formulated is given as follows:

$\{Z_1, Z_2\}$: Z_1 is weakly more important than Z_2.

$\{Z_1, Z_3\}$: Z_1 is moderately more important than Z_3.

$\{Z_2, Z_3\}$: Z_2 is weakly more important than Z_3.

Using the nine point scale [19], and assuming uncertainty in pairwise judgments, the imprecise pairwise comparison matrix can be written by using the formation in (6) as:

$$A = \begin{pmatrix} 1 & [2,3] & [2.5,3.5] \\ [0.4,0.6] & 1 & [1.5,2.5] \\ [0.3,0.4] & [0.4,0.6] & 1 \end{pmatrix} \qquad (19)$$

Then,

$$A_L = \begin{pmatrix} 1 & 2 & 2.5 \\ 0.4 & 1 & 1.5 \\ 0.3 & 0.4 & 1 \end{pmatrix} \quad \text{and} \quad A_U = \begin{pmatrix} 1 & 3 & 3.5 \\ 0.6 & 1 & 2.5 \\ 0.4 & 0.6 & 1 \end{pmatrix}$$

Using the relation in (12), the GP model for determination of weights (in interval form) can be presented as:

$$\text{Minimize } Z = \sum_{i=1}^{2} \sum_{j=1}^{3} (d_{ij}^- + d_{ij}^+)$$

so as to satisfy

$$(0).w_{1U} + 2w_{2U} + 2.5w_{3U} - 2w_{1L} + d_{11}^- - d_{11}^+ = 0$$
$$0.4w_{1U} + (0)w_{2U} + 1.5w_{3U} - 2w_{2L} + d_{12}^- - d_{12}^+ = 0,$$
$$0.3w_{1U} + 0.4w_{2U} + (0)w_{3U} - 2w_{3L} + d_{13}^- - d_{13}^+ = 0,$$
$$(0).w_{1L} + 3w_{2L} + 3.5w_{3L} - 2w_{1U} + d_{21}^- - d_{21}^+ = 0$$
$$0.6w_{1L} + (0)w_{2L} + 2.5w_{3L} - 2w_{2U} + d_{22}^- - d_{22}^+ = 0,$$
$$0.4w_{1L} + 0.6w_{2L} + (0)w_{3L} - 2w_{3U} + d_{23}^- - d_{23}^+ = 0$$
$$w_1^L + w_2^U + w_3^U \geq 1, \quad w_2^L + w_2^U + w_3^U \geq 1, \quad w_3^L + w_1^U + w_2^U \geq 1,$$
$$w_1^U + w_2^L + w_3^L \leq 1, \quad w_2^U + w_1^L + w_3^L \leq 1, \quad w_3^U + w_1^L + w_2^L \leq 1,$$
$$w_1^U \geq w_1^L, \quad w_2^U \geq w_2^L, \quad w_3^U \geq w_3^L. \qquad (20)$$

Using LINGO (ver. 6.0), the result is obtained as:

$$[w_1^U, w_1^L] = [0.5243, 0.5772]; [w_2^U, w_2^L] = [0.2471, 0.3048]; [w_3^U, w_3^L] = [0.11799, 0.1756] \qquad (21)$$

Now, using the derived interval weights the interval goal programming formulation can be expressed as:

Minimize Z=$[0.5243, 0.5772]\eta_1^- + [0.2471, 0.3048]\eta_2^- + [0.1179, 0.1756]\eta_3^-$

so as to satisfy

$(1/230)(70x_1 - 30x_2 - 20) + \eta_1^- - \eta_1^+ = 1$,

$(1/46)(3x_1 + 8x_2 - 20) + \eta_2^- - \eta_2^+ = 1$,

$(1/14)(-4x_1 + x_2 + 18) + \eta_3^- - \eta_3^+ = 1,$

subject to

$$2x_1 + x_2 \geq 8, \quad x_1 + x_2 \geq 5, \quad x_1 - 2x_2 \geq -6, \quad 5x_1 - 2x_2 \leq 18,$$
$$x_1, x_2 \geq 0. \tag{22}$$

Using interval arithmetic, the objective function in interval-valued form can be expressed as:

Minimize Z= $[0.5243\eta_1^- + 0.2471\eta_2^- + 0.1179\eta_3^-,\ 0.5772\eta_1^- + 0.3048\eta_2^- + 0.1756\eta_3^-] = [T_{1L}(\eta^-), T_{1U}(\eta^-)]$ (say)

The least value of T_{1U} is obtained as $T_{1U}^{\bullet} = 0.4064$.

We choose the target interval as $[t_1^L, t_1^U] = [0.100, 0.320]$

Using the proposed procedure defined in (16) the goals expression can be written as:

$$0.5243\eta_1^- + 0.2471\eta_2^- + 0.1180\eta_3^- + \rho_{1L}^- - \rho_{1L}^+ = 0.100,$$
$$0.5772\eta_1^- + 0.3048\eta_2^- + 0.1756\eta_3^- + \rho_{1U}^- - \rho_{1U}^+ = 0.320 \tag{23}$$

Now, the executable GP model can be expressed as:

Minimize Z= $\rho_{1L}^- + \rho_{1U}^+$

so as to satisfy the goal relations in (23), subject to the set of constraints in (22).

Using LINGO (Ver. 6.0), the problem is solved and the solution is obtained as:

$(x_1, x_2) = (6,\ 6)$

with $(Z_1, Z_2, Z_3) = (240, 66, -18)$

Satisfactory solution is achieved here according to the needs and desires of the DM.

Note: If conventional fuzzy weights are used for solving the problem by *minsum* FGP methodology [7], then the resultant solution is

$(x_1, x_2) = (2,4)$ with $(Z_1, Z_2, Z_3) = (20, 38, -4)$

The solution obtained under the proposed approach and conventional FGP approach is shown in the Fig 1.

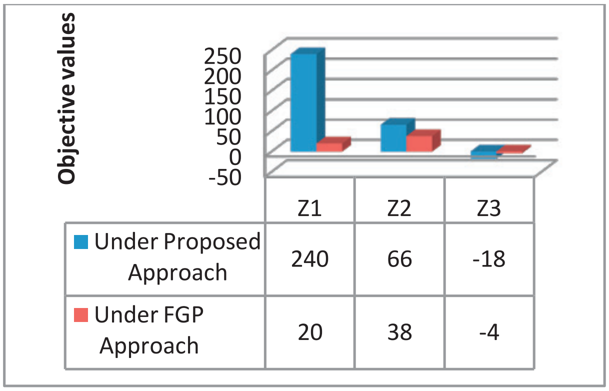

Fig 1. Comparison of the objective values obtained under the proposed approach and FGP approach

The above result shows that the proposed approach is better for achieving the objective values compare to the conventional fuzzy goal programming with fuzzy weights.

8. Conclusion

The main advantage of approach presented here is that the proper weights for achieving goals on the basis of their importance can be assigned in the imprecise decision environment.

The proposed model can be extended to conventional GP and interval programming methodologies to make comprehensive decisions by introducing appropriate weight structure in the decision making environment, which is a problem in future research.

However, it is expected that the proposed approach may open up a new look in the way of solving MODM problems in the current inexact decision environment.

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